sequences & series - 1

4 questions – progressing from ‘accessible’ to ‘discriminating’

1. How many terms are in the following arithmetic sequence?  [ calculator allowed ]
   
   $-9, -2, 5, \ldots, 103$

2. The sum of the first three terms of an arithmetic sequence is 6 and the fourth term is 16.
   Find the first term, $u_1$, and the common difference, $d$, of the sequence.  [ no calculator ]

3. $a, 1, b$ are three consecutive terms of an arithmetic series, and $b, a, \frac{8}{3}$ are the first three terms of an infinite geometric series that has a sum of $S$. Find $a, b$ and $S$.
   [ calculator allowed ]

4. Show that the series $\log_2 x + \log_4 x + \log_{16} x + \cdots$ is geometric and find the sum of the series for infinite terms.  [ no calculator ]
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Answers

1. 17 terms

2. \( u_1 = -5, \ d = 7 \)

3. \( a = -4, \ b = 6, \ S = \frac{18}{5} \)

4. \[
\frac{\log_4 x}{\log_2 x} = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2} = \frac{1}{2} \quad \text{and} \quad \frac{\log_{16} x}{\log_4 x} = \frac{\log_2 x}{\log_2 16} = \frac{\log_2 x}{4} = \frac{1}{2} \Rightarrow \text{series is geometric}
\]

\[
S_\infty = 2\log_2 x \quad [\text{or} \quad \log_2 \left(x^2\right)]
\]