1. Find the sum of the multiples of 3 between 100 and 500

First multiple of 3 over 100 is 102
and greatest multiple of 3 less than 500 is 498

Need to find # of terms \( n \) for sequence 102, 105, 108, ..., 498

\[ a_n = a_1 + (n-1)d \quad \Rightarrow \quad 498 = 102 + 3(n-1) \quad \Rightarrow \quad 3n = 399 \]

\[ n = 133 \]

\[
S_n = \frac{n}{2}(a_1 + a_n) \quad \Rightarrow \quad S_{133} = \frac{133}{2}(102 + 498)
\]

\[
S_{133} = 39,900
\]

2. A sample of radioactive material loses 8% of its mass every month. At this time, the sample contains 800 grams.

(a) Write a function \( A(m) \) that expresses the amount, \( A \), of grams of the sample in terms of the number of months, \( m \), from now.

(b) How many grams of the sample remain 2 years from now?

(a) \[ A(m) = 800 \cdot (0.92)^m \]

(b) \[ A(24) = 800 \cdot (0.92)^{24} \approx 108.143 \]

approximately 108 grams in 2 years
3. Represent each of the following series using sigma notation.

(a) \(-13 - 5 + 3 + \ldots + 83\)

(b) \(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12}\)

(a) Find # of terms
\[83 = -13 + (n-1)8 \Rightarrow 8n = 104 \Rightarrow n = 13\]
and find expression for \(n\)th term: \(a_n = -13 + (n-1)8 = 8n - 21\)

\[-13 - 5 + 3 + \ldots + 83 = \sum_{n=1}^{13} (8n - 21)\]

(b) Series is neither arithmetic nor geometric; 6 terms
General expression for \(n\)th term: \(\frac{1}{2n}\)

\[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \sum_{n=1}^{6} \frac{1}{2n}\]

4. If the coefficient of the \(x^2\) term in the binomial expansion of \((a + 5x)^5\) is 2000, then what is the value of \(a\)?

Expression for general term: \(\binom{5}{r} a^{5-r} (5x)^r\)

For \(x^2\) term, \(r = 2\)

Thus, coefficient of \(x^2\) term is \(\binom{5}{2} a^{3} 5^2 = 2000\)

\[10 \cdot a^3 \cdot 25 = 2000\]

\[a^3 = 8\]

\[a = 2\]
5. The first four terms of an arithmetic sequence are \( a - b, 2a + b + 7 \) and \( a - 3b \), where \( a \) and \( b \) are constants. Find \( a \) and \( b \).

\[
\begin{align*}
\ a - b - 2 & = 2a + b + 7 - (a - b) \\
\ a - b - 2 & = a + 2b + 7 \quad \Rightarrow \quad 3b = -9 \quad \Rightarrow \quad b = -3 \\
\ a - b - 2 & = a - 3b - (2a + b + 7) \\
\ a - b - 2 & = -a - 4b - 7 \quad \Rightarrow \quad 2a + 3b = -5 \\
& \quad \Rightarrow \quad 2a + 3(-3) = -5 \quad \Rightarrow \quad 2a = 4 \\
& \quad \Rightarrow \quad a = 2
\end{align*}
\]

\[ a = 2, \ b = -3 \]

6. A geometric series has a negative common ratio. The sum of the first two terms is 6, and the sum to infinity is 8. Find the common ratio and the first two terms.

\[
\begin{align*}
\text{sum of first two terms:} & \quad a_1 + a_1r = 6 \quad \Rightarrow \quad a_1(1 + r) = 6 \\
S_\infty & = \frac{a_1}{1 - r} = 8 \quad \Rightarrow \quad a_1 = \frac{6}{1 + r} \\
\frac{6}{1 + r} & = 8 \quad \Rightarrow \quad \frac{6}{1 + r} \cdot \frac{1}{1 - r} = 8 \quad \Rightarrow \quad \frac{6}{1 - r^2} = 8 \\
1 - r^2 & = \frac{6}{8} = \frac{3}{4} \quad \Rightarrow \quad r^2 = 1 - \frac{3}{4} \quad \Rightarrow \quad r^2 = \frac{1}{4} \quad \Rightarrow \quad r = \pm \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
a_1 & = \frac{6}{1 + r} = \frac{6}{1 - \frac{1}{2}} = \frac{6}{\frac{1}{2}} = 12 \\
a_2 & = 12 \cdot \left(-\frac{1}{2}\right) = -6 \quad \Rightarrow \quad r = -\frac{1}{2}
\end{align*}
\]

\[ r = -\frac{1}{2}, \ a_1 = 12, \ a_2 = -6 \]
7. Find the term independent of \(x\) in the expansion of \(\left(\frac{1}{x^3} - 2x\right)^6\). The expression for the general term is:
\[
\binom{6}{r} \left(x^{-2}\right)^{6-r} (-2x)^r
\]
Looking at exponents only:
\[-12 + 2r + r = 0\]
\[3r = 12 \implies r = 4\]

\[r = 4 : \quad \binom{6}{4} \left(x^{-2}\right)^2 (-2x)^4 = 15 \cdot 16 x^4 = 240\]

8. The first three terms of an arithmetic sequence have a sum of 24. The first, second and sixth terms of this arithmetic sequence are also consecutive terms of a geometric sequence. Find the first six terms of the arithmetic sequence.

**Arithmetic sequence**

First 3 terms: \(a_1, a_1 + d, a_1 + 2d\)

Sum is 24:
\[a_1 + (a_1 + d) + (a_1 + 2d) = 24 \implies 3a_1 + 3d = 24\]

6th term of arithmetic seq. is \(a_1 + 5d\)

**Geometric sequence**, consecutive terms: \(a_1, a_1 + d, a_1 + 5d\)

\[r = \frac{a_1 + d}{a_1} = \frac{a_1 + 5d}{a_1 + d} \implies (a_1 + d)^2 = a_1 (a_1 + 5d)\]

\[a_1^2 + 2a_1d + d^2 = a_1^2 + 5a_1d \implies 3a_1d - d^2 = 0 \implies d \left(3a_1 - d\right) = 0\]

\[d = 0 \text{ or } d = 3a_1\]

\[a_1 + d = 8 \implies a_1 + 3a_1 = 8 \implies a_1 = 2\]

First 6 terms of arithmetic seq.: \(2, 8, 14, 20, 26, 32\)
9. Find \( n \) if the coefficient of \( x^2 \) in the expansion of \((1 + 2x)^n\) is 112.

Expression for general term: \( \binom{n}{r} (1)^{n-r} (2x)^r \)

\[ X^2 \text{ term} \rightarrow r = 2 \]

Coefficient: \( \binom{n}{2} (2)^2 = 112 \)

\[ \frac{n!}{2!(n-2)!} \cdot 4 = 112 \Rightarrow \frac{n!}{(n-2)!} = 56 \]

\[ \frac{n(n-1)(n-2)(n-3)\ldots}{(n-2)(n-3)(n-4)\ldots} = 56 \Rightarrow n^2 - n - 56 = 0 \]

\[ (n-8)(n+7) = 0 \]

\( n = 8 \) or \( n = -7 \)

**Bonus**

For what values of \( x \) does the infinite series \( 1 + \frac{1}{x+1} + \frac{1}{(x+1)^2} + \cdots \) have a sum?

Geometric series such that \( r = \frac{1}{x+1} \)

Infinite sum exists if \(-1 < r < 1\)

Solve \(-1 < \frac{1}{x+1} < 1\)

\[-1 < \frac{1}{x+1} \]

\[ \frac{1}{x+1} > -1 \Rightarrow X < -2 \text{ or } X > -1 \]

\[ \frac{1}{x+1} < 1 \Rightarrow X < - \text{ or } X > 0 \]

Solution is intersection of these

\[ X < -2 \text{ or } X > 0 \]