Test_3 - sequences & series, binomial theorem

content: sequences & series; binomial expansions; sigma notation; binomial theorem

Part 1 – No GDC on questions 1-5

1. Expand \((3-x)^4\) in ascending powers of \(x\) and simplify your answer. [5 marks]

2. Find the value of \(k\) if \(\sum_{r=1}^{\infty} k \left(\frac{1}{3}\right)^r = 7\). [4 marks]

3. Consider the infinite geometric series \(1 - (1-2x) + (1-2x)^2 - (1-2x)^3 + \ldots\)
   (a) Find the range of values of \(x\) so that the series converges to a finite sum. [5 marks]
   (b) Assuming that \(x\) lies within the range found in (a), find the sum to infinity in terms of \(x\). [2 marks]

4. An arithmetic sequence \(u_1, u_2, u_3 \ldots\) has \(u_1 = 1\) and common difference \(d \neq 0\). Given that \(u_2, u_3\) and \(u_6\) are the first three terms of a geometric sequence
   (a) find the value of \(d\). [4 marks]
   Given that \(u_N = -15\)
   (b) determine the value of \(\sum_{r=1}^{N} u_r\). [3 marks]

5. The sum to infinity of a convergent geometric series is equal to the sum to infinity of the squares of its terms. If the first term of the geometric series is \(\frac{1}{4}\), find its common ratio. [6 marks]

Part 2 – GDC allowed on questions 6-10

6. The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.
   (a) Find the first term and the common difference. [4 marks]
   (b) Find the smallest value of \(n\) such that the sum of the first \(n\) terms is greater than 600. [3 marks]

7. Find the exact value of the constant term in the expansion of \(\left(4x^2 - \frac{3}{2x}\right)^{12}\). [6 marks]

8. Find the sum of all the multiples of 6 between 100 and 500. [5 marks]

9. A geometric sequence has first term \(a\), common ratio \(r\) and sum to infinity 76. A second geometric sequence has first term \(a\), common ratio \(r^3\) and sum to infinity 36. Find \(r\). [7 marks]

10. Find the term independent of \(x\) in the expansion of \(\left(1 - 2x^3\right)^3 \left(\frac{3}{x} + 2x^2\right)^6\). [6 marks]