Activity: transformations of the graph of a function

equipment: TI-Nspire handheld device or TI-Nspire student software
content: transformations of graphs
horizontal/vertical translation, reflection in x or y axis,
horizontal/vertical stretch or shrink, composite transformations

This activity investigates how the graph of a function can be \textit{transformed} by changing the values of parameters (constants) in the expression for the function, and guides you to conjecture about how the value of a particular parameter affects the graph. The relevant transformations include \textit{translation} (slide), \textit{reflection} (flip) and \textit{dilation} (stretch or shrink).

Given a function \( f_1(x) \), you can create a ‘new’ function \( f_2(x) \) by substituting in values for one or more of the parameters \( a, b, c \) or \( d \) such that \( f_2(x) = a \cdot f_1(b(x + c)) + d \). For the ‘original’ function \( f_1(x) \), both \( a \) and \( b \) are equal to one, and both \( c \) and \( d \) are equal to zero. You will investigate how substituting other values (but \( a \neq 0, b \neq 0 \)) transforms the graph of \( f_1(x) \) into the graph of \( f_2(x) \).

You will investigate each of the four parameters separately by using a slider to change the value of one parameter while the values of the other three remain constant and see how this changes the graph.

Although \( f_1(x) \) could be any function, for this activity you will use the cubic function \( f_1(x) = x^2 - \frac{1}{4} x^3 \) as the ‘original’ function and explore the ‘new’ function \( f_2(x) = a \cdot f_1(b(x + c)) + d \).

You will then create a Graphs page with four sliders (see image below) – one for each parameter \( a, b, c \) and \( d \) – to confirm your earlier conjectures about the transformations produced by individually changing the value of each parameter, and to see how a ‘new’ function can be created by a sequence of more than one transformation of the ‘original’ function.
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Part 1 – Initial Set Up

On your TI-Nspire open a new Calculator page. Eventually you will enter \( f_1(x) = x^2 - \frac{1}{4} x^3 \) as our ‘original’ function and a second function as \( f_2(x) = a \cdot f_1(b(x+c)) + d \). At the start of investigating each of the four parameters \( a, b, c \) and \( d \) you want the two functions \( f_1(x) \) and \( f_2(x) \) to be the same. Thus, initially you need to set the values of the parameters to be: \( a = 1, b = 1, c = 0, d = 0 \).

Storing the necessary value for each parameter is a separate entry on the calculator but multiple entries can be included on one entry line by separating each with a colon as shown below. The colon symbol is located in the set of special symbols that are accessed by pressing the \( \text{shift} \) key; and the store command (written as an arrow \( \rightarrow \)) is entered by pressing \( \text{ctrl} \) and then \( \text{var} \).

Be sure to enter multiplication signs in appropriate places. There is no implied multiplication on the TI-Nspire.

Now you are ready to investigate each of the four parameters – one at a time – in a Graphs page.
Activity: transformations of the graph of a function

Part 2 – Investigating parameter \( c \)

\[ f_2(x) = a \cdot f_1(b(x+c)) + d \quad \Rightarrow \quad a = 1, b = 1, d = 0 \quad \text{and value of } c \text{ is changed} \]

Add a Graphs page by pressing \( \text{ctrl} \) and then \( \text{doc} \). Note that the entry line shows \( f_3(x) \). This is because you already defined functions \( f_1(x) \) and \( f_2(x) \) on the previous Calculator page. The Calculator page 1.1 and the Graphs page 1.2 are both pages (1 and 2 respectively) in problem 1 so they will share any variables or functions that have been defined on any page within the problem.

With the cursor positioned here after \( f_3(x) \), press the up arrow twice to see the entry for \( f_1(x) \).

\[ f_1(x) = x^2 - \frac{1}{4} \cdot x^3 \]

If the entry line becomes hidden you can display it again by pressing the tab key.

Notice that the check box to the left of the function is unchecked (that’s why the graph of \( f_1(x) \) is not displayed). By pressing enter the check box becomes checked and the graph of \( f_1(x) \) is shown.

Observe that the color of the graph of \( f_1(x) \) is black. Normally, the first graph is blue (and the 2nd is red) but \( f_1(x) \) is black since it was defined previously.

Change the color of the graph of \( f_1(x) \) to red so that you will be able to distinguish it from the graph of \( f_2(x) \). Do this by using the touchpad to move the pointer (arrow) to touch the graph and the label \( f_1 \) appears. Then press \( \text{ctrl} \) followed by \( \text{menu} \) which causes a menu to appear. Select B:Color, then 1:Line Color and then select red.
**Activity: transformations of the graph of a function**

**Part 2 – Investigating parameter \(c\) (continued)**

- \( f_2(x) = a \cdot f_1(b \cdot (x+c)) + d \quad \Rightarrow \quad a = 1, b = 1, d = 0 \quad \text{and value of} \quad c \quad \text{is changed} \)

Press tab to show the entry line again and press the up arrow once to see the entry for \( f_2(x) \).

Because the current values of the parameters are \( a = 1, b = 1, c = 0, d = 0 \) then \( f_1(x) = f_2(x) \), so the graph of \( f_2(x) \) is on top of the graph of \( f_1(x) \).

**2.1 - Inserting a slider**

Now insert a slider in the Graphs page 1.2 to control the value of \(c\). Press \(\text{menu}\) and select 1:Actions, then B:Insert Slider.

The Slider Settings box will appear. Change the Variable to \(c\) and enter the values for Value, Minimum, Maximum and Step Size shown below.

Press \(\text{tab}\) to move from one setting to the next and press enter after setting Step Size to 0.1.

To move the slider, place the pointer on the slider which causes the pointer to change to an open hand. Press and very briefly hold down the center of the touchpad. The hand will ‘grab’ the slider and by using the touchpad you can move it into a better position – e.g. at the bottom left of the screen. Press \(\text{esc}\) to release. If the blue box does not appear around the slider you can make it appear by pressing \(\text{ctrl menu}\) and select 1:Move. Remember to press \(\text{esc}\) to release your ‘grab’ of the slider.
Activity: transformations of the graph of a function

Part 2 – Investigating parameter $c$ (continued)

- $f_2(x) = a \cdot f_1(b(x + c)) + d \quad \Rightarrow \quad a = 1, b = 1, d = 0$ and value of $c$ is changed

♦ 2.2 – Investigating transformation by changing value of parameter

Place the pointer (arrow) on the slider bar, but not on the marker . It will change to a pointing hand . Press and briefly hold the center of the touchpad . The value of $c$ immediately changes to the value being pointed to by the hand . Now you can easily change the value of $c$ by pressing the left or right arrows on the touchpad. You can also change the value of $c$ by grabbing the marker and moving it with the touchpad, but it’s easier to control the value of $c$ with the arrow buttons.

Explore the effect that changing the value of $c$ has on the graph of $f_2(x)$. Remember $a = 1, b = 1, d = 0$.

When $c = 0$, $f_2(x) = x^2 - \frac{1}{4} x^3$, i.e. the ‘original’ function whose graph is shown in red. What happens to the graph of $f_2(x)$ when $c$ is changed to 3?

That is, what transformation occurs to the graph of a function $f$ when $f(x)$ is changed to $f(x + 3)$?

Answer the question in the box below.

Part 2: $f(x) \Rightarrow f(x + c)$ How does the value of $c$ affect the graph of $f(x)$?

Parameters $d$, $a$ and $b$ are investigated in following sections.
Activity: transformations of the graph of a function

Part 3 – Investigating parameter $d$

Repeat parts 2.1 (inserting a slider - pg.4) and 2.2 (changing value of parameter - pg.5) for parameter $d$. Be sure to have the value of $c$ set to zero.

Part 3: $f(x) \Rightarrow f(x) + d$  How does the value of $d$ affect the graph of $f(x)$?

Part 4 – Investigating parameter $a$

Repeat parts 2.1 (inserting a slider - pg.4) and 2.2 (changing value of parameter - pg.5) for parameter $a$. Be sure to have the value of both $c$ and $d$ set to zero.

Part 4: $f(x) \Rightarrow a \cdot f(x)$  How does the value of $a$ affect the graph of $f(x)$?
Activity: transformations of the graph of a function

Part 5 – Investigating parameter $b$

Repeat parts 2.1 (inserting a slider – pg.4) and 2.2 (changing value of parameter – pg.5) for parameter $b$. Be sure to have the value of both $c$ and $d$ set to zero, and the value of $a$ set to one.

Part 5: $f(x) \Rightarrow f(b \cdot x)$ How does the value of $b$ affect the graph of $f(x)$?

Part 6 – Changing more than one parameter

Now with all four sliders on the Graphs page you can see the overall effect created when more than one of the parameters $a, b, c$ and $d$ are changed from their initial values of $a = 1, b = 1, c = 0, d = 0$. 